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Uncovering the Secrets Behind Enumeration Rules: The Art of Combinatorics Thinking

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Abstract. This research aims to analyze students' combinatorial thinking skills after using learning worksheets on enumeration rules. This research aims to provide solutions by applying learning materials to improve students' combinatorial thinking skills in solving math problems. The research method used is qualitative research. The research technique in this case is to use data triangulation. The purpose of triangulation is to increase the validity and reliability of research findings by confirming or complementing findings from various sources or perspectives. The results in this study are the diversity of the fulfillment of indicators in the ability to think combinatorics, including students with the S-1 category can fulfill all indicators of combinatorics thinking ability. Students in the S-2 category only fulfill 1 indicator of combinatorics thinking ability. Students in the S-3 category could not fulfill all the indicators of combinatorics thinking ability

Keywords: Enumeration Rules, Combinatorics thinking skills, Math Learning.

I. INTRODUCTION

An enumeration rule is a rule used to count or determine the number of possible outcomes of a particular experiment or event. The Enumeration Rule is used in selecting a model that is specifically suited to learners' knowledge to obtain several possibilities according to the underlying mechanism (Hara & Ishihata, 2018). The Rule of Enumeration is one of the materials in mathematics subjects discussed at the secondary school level in Indonesia. The function of the enumeration rule is as the basis of an instrument to learn other materials that are mostly combinatorial in nature (Nathania, 2021). Developing combinatorial skills is very important for educators, as it increases professionalism and aids in training future teachers (Izza et al., 2023).

Combinatorial thinking is a special aspect of mathematical thinking (Rezaie & Gooya, 2011). Combinatorial thinking is the ability of learners to consider all possible alternatives in a given situation and construct solutions and solve problems by combining several factors and concluding as a result of their combination (Inhelder & Piaget, 1958; Pásztor & Csapó, 2012). The development of combinatorial thinking skills improves learners' probabilistic knowledge and understanding, enabling them to evaluate the credibility of hypotheses and make informed decisions (Savenkov et al., 2021). Overall, the development of combinatorial thinking skills equips learners with the necessary tools to think critically, solve problems independently, and make informed decisions in mathematics learning.

The researcher in this case is a mathematics teacher at the senior high school level in Indonesia. The enumeration rules material discussed at this level is related to probability (Mone, 2021). Students are trained to think critically in solving problems related to everyday life. Embedding concepts from those around us is expected to make students have provisions so that in the future they can solve problems, design and produce new works and innovations. Findings based on facts in the field state that there are students who have difficulty in solving problems on enumeration rules material. In addition, learners may face ontogenic barriers, such as fear and lack of understanding of prerequisite material, as well as epistemological barriers, where learners struggle to apply algebraic thinking to story problems (Sidik et al., 2021). Furthermore, learners may face difficulties in answering random questions, understanding teacher explanations, and external factors that hinder cognitive development (Oktaviani et al., 2023). To overcome these challenges, combinatorial thinking skills can be a solution. By developing critical thinking skills and understanding mathematical concepts, learners can improve their cognitive development (Velez et al., 2023). The main objective in this study is to analyze students' combinatorial thinking skills after using the learning worksheet. This research aims to provide solutions by applying learning materials to improve students' combinatorial thinking skills in solving problems.

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II. METHODS

The research method used is qualitative research. The research technique in this case is to use data triangulation. The purpose of triangulation is to increase the validity and reliability of research findings by confirming or complementing findings from various sources or perspectives (Rifa'i, 2023). The final result of qualitative research is in the form of an information formulation or thesis statement (Susanto et al., 2020). The research techniques used are interviews, observations, and content analysis, which can be used to obtain in-depth information about students' experiences in developing combinatorial thinking skills and understanding how students combine their thinking to solve problems and how they communicate and articulate their understanding (Delmas & Giles, 2023).

According to Khunaifi & Susanah (2023) there are levels in combinatorics thinking skills cited from Rezaie and Gooya (2011) including:

1. Level 1 is that students can investigate a case of mathematical problems and statements. the indicators at this level are:
   a. Able to explain the purpose and objectives of the problem
   b. Able to determine the case or information from the problem
2. Level 2 is that students can be confident in solving problems that are done in their own way. Indicators at this level are:
   a. Visualizing alternative answers to the given problem
3. Level 3 is that students can draw conclusions and generalize solutions. Indicators at this level are:
   a. Able to solve the first problem
   b. Able to draw conclusions from the overall answer
   c. Able to discover new concepts
4. Level 4 is that students can validate conclusions by working on different problems but the same in the context of the solution. The indicators of this level are:
   a. Able to solve the second problem
   b. Able to give examples of other similar combinatoric problems
   c. Able to apply similar solving concepts to other combinatoric problems that have been encountered.

Instruments

The instrument used in this research is a form of questions related to the rules of enumeration to measure the ability to think mathematical combinatorics and deepened through interview techniques.

Table 1 Question Instrument

No.	Question
1.	Find the number of 4-letter arrangements of the letters in the set { p, q, r, s, t } if their order is observed?
2.	The number of ways to choose the chairman, secretary and treasurer from 4 candidates including Doni, Dani, Dina and Dini is ....
3.	From the 12 candidates for the board of a foundation, 3 people will be chosen to hold the positions of chairman, treasurer and secretary. The number of possible board members is ....
4.	The number of numbers consisting of 2 different numbers composed of the numbers 1, 3, 5, 7 is ....
5.	A shirt is designed in two colors. If there are 4 fabric colors available including yellow, purple, green and blue. How many models can be made?

According table 1 explain about questions about the enumeration rules used in this study are 5 questions which will then be analyzed and deepened using interview techniques.

III. RESULTS AND DISCUSSION

The results in this study are in the form of student answers to analyze the combinatorics thinking ability of students. The number of research subjects was 131 students from 5 study groups who were 12th grade students of SMK PUI GEGESIK with Multimedia and Motorcycle Engineering majors. Data acquisition regarding combinatorics thinking ability is taken from students' answers in working on problems related to the rules of enumeration.

The result of S-1's answer

The following are the results of the answers done by S-1

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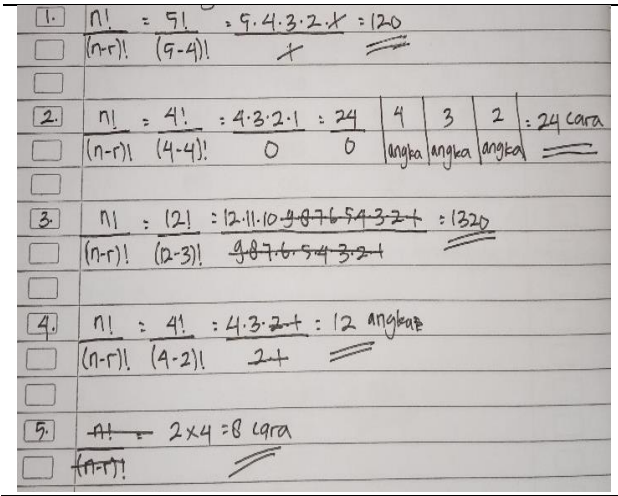

1. $\frac{n!}{(n-r)!} = \frac{5!}{(5-4)!} = \frac{5.4.3.2.1}{1} = 120$

2. First way

$$\frac{n!}{(n-r)!} = \frac{4!}{(4-4)!} = \frac{4.3.2.1}{0} = \frac{24}{0}$$

Second way

4 numbers	3 numbers	2 numbers

= 24 ways

3. $\frac{n!}{(n-r)!} = \frac{12!}{(9-3)!} = \frac{12.11.10.9.8.7.6.5.4.3.2.1}{9.8.7.6.5.4.3.2.1} = 1320$
4. $\frac{n!}{(n-r)!} = \frac{4!}{(4-2)!} = \frac{4.3.2.1}{2.1} = 12\ numbers$
5. $\frac{\cancel{n!}}{\cancel{(n-r)!}} - 2 \times 4 = 8\ ways$

Figure 1 S-1 answer results

Based on Figure 1 shows that in problem number 1 S-1 can explain the purpose and objectives of the problem and can understand problem number 1 well and solve the problem according to his knowledge. In number 2 S-1 made a mistake in calculating 0! But it was clarified again by writing the second method that produced the same answer. wrote the answer according to what was known in the problem and S-1 could solve the problem but in working on problem number 5 S-1 experienced doubts that previously wanted to write the same method as in number 1 to number 4 so that in number 5 S-1 made a mistake. this is clarified in the following interview:

Researcher: "What do you understand about the first problem?"

S-1 : "In the problem there is a set whose members are the letters P, Q, R, S and T and then they will be arranged in a different order."

Researcher: "How did you solve it?

S-1 : "Using the permutation rule mom, where n = 5 and r = 4 then input into the formula which is n! Divided by (n-r)! So 5! divided by (5-4)! So the result is 120"

Researcher: "How did you solve problem number 2?"

S-1 : "The method is the same as problem number 1 using the permutation rule where n = 4 and r = 4 then 4! divided by (4-4)! So the result is 4! divided by 0! But I don't know the result of 0! Then I used another way, namely by logic, for the position as chairman there are 4 possibilities, as secretary there are 3 possibilities and treasurer there are 2 possibilities then multiplied all the results are 24."

Researcher : "How did you solve problem number 3?"

S-1 : "For problem number 3 I used the permutation rule with n = 12 and r = 3 then 12! Divided by (12-3)! So that 12! Divided by 9. So the result is 1,320."

Researcher: "How did you solve problem number 4?"

S-1 : "For problem number 4 use the same method with n = 4 and r = 2 then 4! Divided by (4-2)! Then 4! Divided by 2. So the result is 12."

Researcher : "How did you solve problem number 5?"

S-1 : "For problem number 5, use the multiplication rule so that 2 x 4 = 8."

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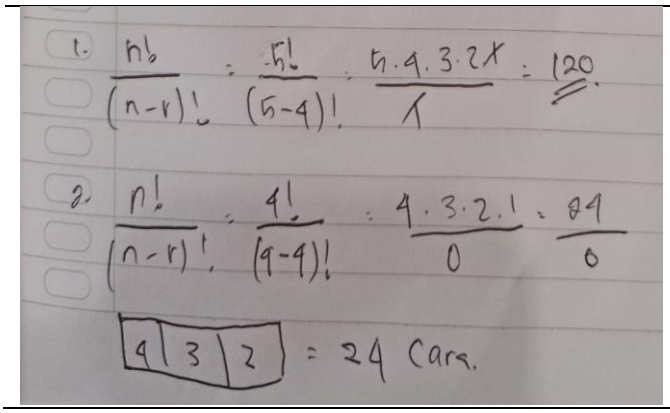
S-2 answer result



1. $\frac{n!}{(n-r)!} = \frac{5!}{(5-4)!} = \frac{5.4.3.2.1}{1} = 120$

2. First way

$$\frac{n!}{(n-r)!} = \frac{4!}{(4-4)!} = \frac{4.3.2.1}{0} = \frac{24}{0}$$

Second way

4	3	2

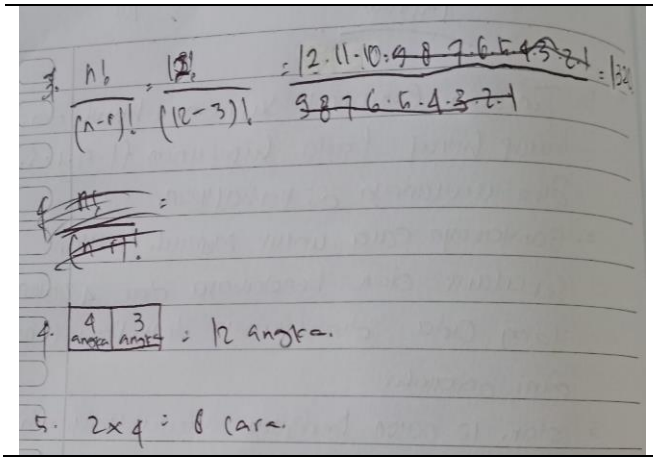
= 24 ways

Figure 2 S-2's answer number 1 to 2

Based on Figure 2, it shows that S-2 can write the answer according to what is known in the problem, in number 1, S-2 can write the answer and in problem number 2, S-2 can solve it even though there is an error when calculating the result of (4-4)! But it was clarified again by using the second method which showed the same result. This is explained in the following interview.

Researcher : "How did you solve problem number 1?"
S-2 : "Using the permutation rule with n = 5 and r = 4, then 5! Divided by (5-4)! So the result is 120."

Researcher : "How did you solve problem number 2?"
S-2 : "The method is the same as number 1, namely using the permutation rule with n = 4 and r = 4 then how 4! Divided by (4-4)! So the result is 24 divided by 0 but I'm not sure then I tried the second way using the place filling rule with 4 x 3 x 2 equals 24."



3. $\frac{n!}{(n-r)!} = \frac{12!}{(12-3)!} = \frac{12.11.10.9.8.7.6.5.4.3.2.1}{9.8.7.6.5.4.3.2.1} = 132$

4.

4 numbers	3 numbers

=12 numbers

5. 2 x 4 = 8 ways

Figure 3 Results of S-2's answers to questions number 3 to 5

Figure 3 shows that S-2 can solve problem number 3 in a structured way. However, in working on problems No. 4 and 5, S-2 admitted that he was confused in answering the questions so he worked by multiplying the number of all objects contained in the problem. This was explained in the following interview.

Researcher : "How did you solve problem number 3?"
S-2 :" I used the permutation rule too with n = 12 and r = 3 then 12! Divided by (12-3)! Then 12! Divided by 9! So the result is 1320."

Researcher: "How did you solve problem number 4?"
S-2: "I used the rule of filling the place then 4 multiplied by 3 the result is 12."

Researcher: "Where is the number 4 from?"
S-2: "From all the objects, ma'am."

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Researcher : "Where did the number 3 come from?"
S-2: "I don't know, I'm confused, but the result is 12."

Researcher : "How did you solve problem number 5?"
S-2 : "I multiply 2 times 4 equals 8."

S-3 answer result

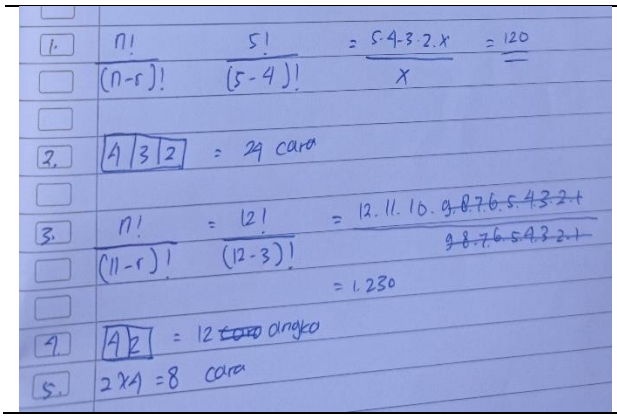

Figure 4 Result of S-3's answer

Based on Figure 4 shows that S-3 can solve problem number 1 in a structured manner. In problem number two, he only wrote the answer briefly. In problem number three, S-3 can write the solution systematically but is wrong in calculating the final result and in problem numbers 4 and 5, S-3 cannot write the answer systematically. This is explained in the following interview.

Researcher : "How did you solve problem number 1?"
S-3 : "I used the permutation rule with n = 5 and r = 4 then 5! Divided by (5-4)! So the result is 120."

Researcher : "How did you solve problem number 2?"
S-3 : "The trick is by multiplying 4 x 3 x 2 the result is 24."

Researcher : "How did you solve problem number 3?"
S-3 : "I used the method like number 1 by entering the numbers 12 and 3 so that the result is 1230."

Researcher : "How did you solve problem number 4?"
S-3 : "The method I used by multiplying 4 x 3 the result is 12."

Researcher: "How did you solve problem number 5?"
S-3 : "by multiplying 2 x 4 the result is 8."

Discussion

Based on the following research results show that S-1 can identify problems characterized by students can understand the problems contained in the problem, can re-understand the problems found marked by S-1 can understand the problems of each question even though the context is different, can describe the problem systematically marked by understanding which is included and referred to in the problem, and change the problem into another combinatorial problem.
Based on the following research results, it shows that S-2 can identify problems marked by students can understand the problems contained in the problem, cannot re-understand the problems found as a whole marked by S-2 cannot understand the problems in questions 4 and 5 even though the answer to number 4 is actually correct, cannot explain the problem systematically and as a whole marked by not understanding what is included and what is meant in questions number 4 and 5, and cannot turn problems into other combinatorial problems in questions number 4 and 5.

Based on the following research results, it shows that S-3 cannot identify the problem which is characterized by students not being able to understand the problems contained in the problem, cannot re-understand the problem found as a whole marked by S-3 not being able to understand the problem in questions 2, 4 and 5, cannot explain the problem systematically and as a whole marked by not understanding what is included and what is meant in questions number 2, 4 and 5, and cannot change the problem into another combinatorial problem in questions number 2, 4 and 5.

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Based on the results of the research above, it is in line with previous research conducted by (Damayanti, 2021; Wahyuni et al., 2023) which shows that students are said to have combinatorial thinking skills if they fulfill the four indicators of combinatorial thinking skills. Combinatorial thinking skills include being able to identify problems, being able to understand problems, being able to understand problems and being able to describe problems systematically and being able to turn problems into combinatorial problems. This is in line with the results of research conducted by Susanti & Taufik (2020) showing that students who have high learning independence will show a positive attitude that is reflected in their learning and have resilience and continue to try to solve problems. This is also in line with research conducted by Oros & Petechuk (2022) that students with high levels of learning independence have the best development of creative thinking skills. This is because these students are accustomed to interpreting a problem into an explicit awareness in seeking new knowledge, methods and knowledge in problem solving.
It is also explained by Abdurakhimovich (2023) stating that students who have combinatorial thinking skills are Students who are able to analyze problems, determine which combinatorial methods can be used, and apply them to find solutions; Creating combinatorial formulas. Students should be able to express combinatorial problems as formulas using appropriate combinatorial objects and operations; Working with combinatorial models or using combinatorial models is another way to develop combinatorial thinking. Students can use various models such as trees, tables, diagrams or probability grids to visualize combinatorial problems and better understand their solutions; Solving problems related to combinatorial applications in real life helps students see the practical applications of combinatorial thinking.

IV. CONCLUSIONS

Students with S-1 category can fulfill all indicators of combinatorics thinking ability including identifying problems, understanding problems, describing problems systematically and turning problems into combinatorics problems. Students in the S-2 category only fulfill 1 indicator of combinatoric thinking ability which is able to identify problems, while the indicators that are not fulfilled are unable to understand the problem, describe the problem systematically and turn the problem into a combinatoric problem. Students in the S-3 category could not fulfill all indicators of combinatorics thinking ability including identifying problems, understanding problems, describing problems systematically and turning problems into combinatorics problems.

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